| **Test Content Categories**  | **How well do I know the content? (scale 1–5)** | **What resources do I have/need for this content?** | **Where can I find the resources I need?** | **Dates I will study this content** | **Date completed** |
| --- | --- | --- | --- | --- | --- |
| 1. Specialized Mathematics Knowledge for Teaching
 |  |  |  |  |  |
| 1. Instruction
 |  |  |  |  |  |
| 1. Knows how to evaluate explanations, justifications, and definitions
 |  |  |  |  |  |
| 1. Identifies valid explanations of mathematical concepts (e.g., explaining why a mathematical idea is considered to be true), models, representations, or procedures
 |  |  |  |  |  |
| 1. Evaluates or compares explanations and justifications for their validity, generalizability, coherence, or precision, including identifying flaws in explanations and justifications
 |  |  |  |  |  |
| 1. Determines the changes that would improve the validity, generalizability, coherence, or precision of an explanation or justification
 |  |  |  |  |  |
| 1. Evaluates definitions or other mathematical language for validity, generalizability, precision, usefulness in a particular context, or support of key ideas
 |  |  |  |  |  |
| 1. Knows how to evaluate problems, tasks, questions, examples, and procedures
 |  |  |  |  |  |
| 1. Identifies problems, tasks, or questions that fit a particular structure, address the same concept, demonstrate desired characteristics, or elicit particular student thinking
 |  |  |  |  |  |
| 1. Identifies parallel problems that systematically vary in complexity in order to differentiate and provide appropriate challenge
 |  |  |  |  |  |
| 1. Sequences problems, tasks, or examples based on information about learning trajectories or standards progressions
 |  |  |  |  |  |
| 1. Evaluates the usefulness of problems, tasks, or examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
 |  |  |  |  |  |
| 1. Identifies examples or questions that support particular strategies or address particular student questions, misconceptions, or challenges with content
 |  |  |  |  |  |
| 1. Identifies nonexamples or counterexamples that highlight a mathematical distinction or demonstrate why a student conjecture is incorrect or partially incorrect
 |  |  |  |  |  |
| 1. Evaluates procedures for working with mathematics content in terms of validity, appropriateness, or rigor, or to identify special cases in which the procedure might be problematic
 |  |  |  |  |  |
| 1. Knows how to evaluate the use of representations and tools (e.g., models, manipulatives, technologies) to support a particular learning goal
 |  |  |  |  |  |
| 1. Evaluates representations (i.e., verbal, visual, physical, contextual, symbolic) in terms of validity, generalizability, usefulness for supporting students’ understanding, or fit to the concept, calculation, etc., to be represented
 |  |  |  |  |  |
| 1. Evaluates how representations (i.e., verbal, visual, physical, contextual, symbolic) have been used to show particular ideas, relationships between ideas, processes, or strategies
 |  |  |  |  |  |
| 1. Evaluates the appropriateness of technologies (e.g., virtual manipulatives, interactives, software) for supporting key ideas in different instructional settings (e.g., face-to-face, online, blended)
2. Student Reasoning
 |  |  |  |  |  |
| 1. Knows how to evaluate student reasoning
 |  |  |  |  |  |
| 1. Identifies likely misconceptions about or partial understanding of particular mathematics content and full engagement in mathematical processes and practices
 |  |  |  |  |  |
| 1. Identifies how new mathematics content and practices can build on or connect to students’ prior knowledge
 |  |  |  |  |  |
| 1. Evaluates or compares student work (e.g., solutions, conjectures, explanations, justifications, generalizations, representations) in terms of validity, generalizability, coherence, or precision
 |  |  |  |  |  |
| 1. Evaluates student work to identify the use of a particular concept, idea, or strategy, and purposefully sequences the presentation of student work in class discussions
 |  |  |  |  |  |
| 1. Evaluates whether a counterargument provides an accurate critique of a given student conjecture, explanation, justification, or generalization
 |  |  |  |  |  |
| 1. Identifies how a student’s reasoning would replicate across similar problems
 |  |  |  |  |  |
| 1. Identifies different pieces of student work that demonstrate the same or similar reasoning
 |  |  |  |  |  |
| II. Pedagogical Knowledge and Instructional Leadership |  |  |  |  |  |
| 1. Pedagogical Knowledge for Teaching Mathematics
 |  |  |  |  |  |
| 1. Understands how child, preadolescent, and adult learning and development affect the mathematical learning environment
 |  |  |  |  |  |
| 1. Identifies ways to draw on each and every learner’s mathematical strengths to create inclusive social learning contexts that engage each and every learner in discussions and mathematical explorations in order to motivate and extend learning opportunities that connect to each and every learner’s experience
 |  |  |  |  |  |
| 1. Identifies how to demonstrate and encourage equitable and ethical treatment of each and every learner and support each and every learner in achieving high expectations
 |  |  |  |  |  |
| 1. Uses instructional formats (e.g., whole group, small group, partner, individual) skillfully and flexibly in support of learning goals and in consideration of various settings (e.g., face-to-face, online, blended)
 |  |  |  |  |  |
| 1. Identifies ways to support the equitable learning of mathematics by embracing and purposefully incorporating diversities of the classroom and school—cultural, racial, ethnic, ability, linguistic, gender, socioeconomic, developmental, etc.; uses this knowledge to motivate and extend learning opportunities
 |  |  |  |  |  |
| 1. Identifies ways to provide each and every learner with opportunities to make connections between mathematics and other content areas, everyday life, and the workplace
 |  |  |  |  |  |
| 1. Identifies ways to facilitate each and every learner’s engagement in productive struggle
 |  |  |  |  |  |
| 1. Understands equitable curriculum and assessment practices
 |  |  |  |  |  |
| 1. Identifies connections between mathematical concepts as well as the developmental progressions within these mathematical concepts
 |  |  |  |  |  |
| 1. Uses standards progressions and learning trajectories to organize and deliver instruction that is developmentally appropriate and responsive to individual learners
 |  |  |  |  |  |
| 1. Uses multiple strategies (e.g., asking probing questions, listening to learners) to assess mathematical knowledge and to understand thinking processes
 |  |  |  |  |  |
| 1. Determines the suitability of mathematics curricula and teaching materials (e.g., curricular resources, technology, manipulatives) and selects, uses, and adapts those materials appropriately for particular learning goals
 |  |  |  |  |  |
| 1. Identifies the different formats, purposes, uses, and limitations of various types of assessment of student learning in order to choose, design, or adapt assessment tasks
 |  |  |  |  |  |
| 1. Uses the formative assessment process (administer a formative assessment task, analyze responses to the task, and determine appropriate actions based on this analysis) in order to inform teaching and benefit learning
 |  |  |  |  |  |
| 1. Analyzes formative and summative assessment results, makes appropriate interpretations, and communicates results to appropriate and varied audiences
 |  |  |  |  |  |
| 1. Selects and uses strategies to provide and manage timely, targeted, and effective feedback (e.g., teacher to student, student to teacher, student to student, among peers)
 |  |  |  |  |  |
| 1. Instructional Leadership
 |  |  |  |  |  |
| 1. Knows how to provide instructional leadership in mathematics
 |  |  |  |  |  |
| 1. Identifies ways to promote and support a rigorous district instructional program based on research-supported best practices regarding curriculum, instruction, technology, and assessment
 |  |  |  |  |  |
| 1. Selects appropriate and effective methods for communicating professionally with educational stakeholders about students, curriculum, instruction, use of technology, and assessment
 |  |  |  |  |  |
| 1. Demonstrates knowledge of educational structures and policies that affect equitable access to quality mathematics instruction
 |  |  |  |  |  |
| 1. Identifies ways to advocate for the rights and needs of each and every learner to secure resources and promote academic advancement
 |  |  |  |  |  |
| 1. Identifies strategies for conferring and collaborating with stakeholders to develop, implement, evaluate, and improve mathematics programs (e.g., curriculum, instruction, professional development, parent/guardian training for at‑home support for students)
 |  |  |  |  |  |
| 1. Identifies professional development needs, and selects and uses strategies to plan, develop, implement, and evaluate professional development programs at the school or district level
 |  |  |  |  |  |
| 1. Identifies ways to use professional development (e.g., mentoring, coaching, peer‑teaching, workshops) to facilitate appropriate research‑supported, standards‑based mathematics instruction and to promote the use of instructional methods supported by research
 |  |  |  |  |  |
| 1. Identifies ways to support teachers in systematically reflecting and learning from practice (e.g., one-on-one observation, coaching cycle, video analysis, lesson study)
 |  |  |  |  |  |
| 1. Applies skills and strategies for mentoring, coaching, and consultation in the development, implementation, and evaluation of an effective mathematics program
 |  |  |  |  |  |
| 1. Identifies differences among coaching moves (e.g., telling, direct guidance, invitational guidance)
 |  |  |  |  |  |
| 1. Identifies differences among roles on a continuum of instructional leadership (i.e., coach versus administrator)
 |  |  |  |  |  |
| 1. Identifies ways to translate research into practices that teachers can use
 |  |  |  |  |  |
| 1. Selects and uses strategies to determine the impact of daily and annual contributions as an elementary math specialist to mathematics teaching and learning, and uses efficacy data to advocate for the role
 |  |  |  |  |  |
| **Mathematics Content** |  |  |  |  |  |
| Elementary Math Specialists have a deep and extensive knowledge of the foundational concepts of pre-K through 6 mathematics. Thus, an Elementary Math Specialist will not only have knowledge of the following mathematics content but will also possess deep conceptual understanding and flexible procedural understanding of these topics, understand key connections among these topics, including ways they build upon and support one another within and between grade levels, and understand how to help students and other stakeholders develop the knowledge and skills described in these topics. |  |  |  |  |  |
| Note that the percentages given for the four domains of mathematics content (i.e., Numbers and Operations, Equations and Expressions, Measurement and Geometry, Statistics) represent an approximate percentage of the questions in Categories I and II that assess mathematics content. For example, of the items in Categories I and II that assess mathematics content, 15% will be about Measurement and Geometry. |  |  |  |  |  |
| 1. Numbers and Operations (60%)
 |  |  |  |  |  |
| 1. Understands counting both conceptually and procedurally
 |  |  |  |  |  |
| 1. Counts and skip counts whole numbers between 0 and 1,000
 |  |  |  |  |  |
| 1. Counts on, starting with any whole number
 |  |  |  |  |  |
| 1. Connects counting to cardinality
 |  |  |  |  |  |
| 1. Demonstrates understanding of one-to-one correspondence between numbers and objects being counted
 |  |  |  |  |  |
| 1. Subitizes (recognizes small quantities by sight) conceptually and perceptually
 |  |  |  |  |  |
| 1. Identifies relationships between counting and the concept of larger and smaller numbers (i.e., that sets with higher counts are larger than sets with smaller counts)
 |  |  |  |  |  |
| 1. Understands operations with whole numbers both conceptually and procedurally
 |  |  |  |  |  |
| 1. Demonstrates understanding of representations (i.e., verbal, visual, physical, contextual, symbolic) of addition, subtraction, multiplication, and division and connections among these representations
 |  |  |  |  |  |
| 1. Solves mathematical and real‑world problems involving the four operations using multiple approaches and determines the reasonableness of results within the context of a given problem
 |  |  |  |  |  |
| 1. Uses properties of operations (e.g., commutative, associative, distributive) to solve mathematical and real-world problems in multiple ways
 |  |  |  |  |  |
| 1. Solves mathematical and real‑world problems using basic concepts of number theory, including prime and composite numbers, factors, and multiples
 |  |  |  |  |  |
| 1. Understands place value and decimals both conceptually and procedurally
 |  |  |  |  |  |
| 1. Demonstrates a conceptual understanding of the value and relationships among digits in numbers
 |  |  |  |  |  |
| 1. Compares multidigit and decimal numbers
 |  |  |  |  |  |
| 1. Compares, orders, classifies, and represents real numbers
 |  |  |  |  |  |
| 1. Rounds multidigit and decimal numbers
 |  |  |  |  |  |
| 1. Composes and decomposes multidigit numbers into groupings and explains why grouping and ungrouping are helpful in performing operations on multidigit and decimal numbers
 |  |  |  |  |  |
| 1. Uses drawings and objects such as manipulatives to represent place value, relating these drawings and objects to numerical equations and written descriptions
 |  |  |  |  |  |
| 1. Understands fractions and operations with fractions both conceptually and procedurally
 |  |  |  |  |  |
| 1. Identifies and represents fractions as part-whole relationships, as multiples of unit fractions, as numbers, as division (e.g., ), and as ratios, moving back and forth flexibly among these conceptualizations
 |  |  |  |  |  |
| 1. Describes characteristics of fractions that are less than one, equal to one, and greater than one
 |  |  |  |  |  |
| 1. Partitions shapes into equal shares and recognizes that equipartitioning is a building block for understanding fractions as part-whole relationships
 |  |  |  |  |  |
| 1. Identifies and represents equivalent fractions
 |  |  |  |  |  |
| 1. Uses a variety of strategies for comparing fractions to other fractions or decimal numbers, where there are two or more numbers being compared
 |  |  |  |  |  |
| 1. Performs operations such as addition, subtraction, multiplication, and division with fractions as well as with fractions and whole numbers, recognizing and using different strategies for these operations, and building intuition about how the operations work (e.g., recognizing that multiplying a whole number by a fraction that is less than one makes the product smaller)
 |  |  |  |  |  |
| 1. Understands ratios, proportions, and percents both conceptually and procedurally
 |  |  |  |  |  |
| 1. Applies the concepts of ratios and unit rates to describe relationships between two quantities and to solve problems
 |  |  |  |  |  |
| 1. Converts flexibly among equivalent decimals, fractions, and percents
 |  |  |  |  |  |
| 1. Identifies and represents proportional relationships
 |  |  |  |  |  |
| 1. Uses proportional relationships to solve ratio, percent, and scaling problems
 |  |  |  |  |  |
| 1. Equations and Expressions (15%)
 |  |  |  |  |  |
| 1. Understands equations and expressions both conceptually and procedurally
 |  |  |  |  |  |
| 1. Recognizes what it means for algebraic terms, expressions, and equations to be considered equivalent, how the equal sign is used to represent relational equivalence, and that equations maintain their equivalence status under certain algebraic manipulations
 |  |  |  |  |  |
| 1. Follows the standard order of operations (including the use of parentheses and the distributive property of multiplication over addition), and uses properties of operations to evaluate and manipulate algebraic expressions, equations, and formulas
 |  |  |  |  |  |
| 1. Uses different interpretations of the word “variable” (e.g., the ideas of quantities that are unknown, which underlies understanding of solving equations, and quantities that vary, which can be connected to patterns and will support later understanding of functional relationships) in different situations
 |  |  |  |  |  |
| 1. Translates between verbal statements and algebraic expressions or equations
 |  |  |  |  |  |
| 1. Determines whether equations are true, identifies the missing values that would make them true, solves equations using the four operations, and solves relational statements
 |  |  |  |  |  |
| 1. Interprets solutions of multistep one-variable linear equations and inequalities
 |  |  |  |  |  |
| 1. Uses linear relationships represented by equations, tables, and graphs to solve problems
 |  |  |  |  |  |
| 1. Uses the less-than and greater‑than relational symbols ( , ) to compare quantities
 |  |  |  |  |  |
| 1. Uses formulas to determine unknown quantities
 |  |  |  |  |  |
| 1. Knows how to recognize and represent patterns and functions both conceptually and procedurally
 |  |  |  |  |  |
| 1. Identifies, extends, describes, or generates number, shape, and contextual patterns
 |  |  |  |  |  |
| 1. Identifies relationships between the corresponding terms of two numerical patterns (e.g., find a rule for a function table)
 |  |  |  |  |  |
| 1. Determines if a function is linear or nonlinear
 |  |  |  |  |  |
| 1. Identifies the independent variable and dependent variable of a function
 |  |  |  |  |  |
| 1. Develops a function—represented by a graph, equation, or table—to model a given set of conditions
 |  |  |  |  |  |
| 1. Evaluates and interprets mathematical models (e.g., graph, equation, table) in context
 |  |  |  |  |  |
| 1. Measurement and Geometry (15%)
 |  |  |  |  |  |
| 1. Understands measurement both conceptually and procedurally
 |  |  |  |  |  |
| 1. Recognizes which attributes of objects are measurable and uses common measurable attributes to compare two objects
 |  |  |  |  |  |
| 1. Selects and uses appropriate measurement tools and standard and nonstandard units of measurement (e.g., length, area, volume, angle)
 |  |  |  |  |  |
| 1. Calculates and estimates perimeter, area, volume, and measurements of angles in mathematical and real-world problems, including composed shapes
 |  |  |  |  |  |
| 1. Uses nets that are made of rectangles and triangles to determine the surface area of three-dimensional figures
 |  |  |  |  |  |
| 1. Determines how changes to dimensions affect measures of area and volume
 |  |  |  |  |  |
| 1. Identifies relative sizes of United States customary units and metric units and converts units within each system
 |  |  |  |  |  |
| 1. Understands geometry both conceptually and procedurally
 |  |  |  |  |  |
| 1. Identifies and classifies two‑dimensional and three‑dimensional figures and their attributes
 |  |  |  |  |  |
| 1. Composes and decomposes shapes
 |  |  |  |  |  |
| 1. Draws shapes based on specific attributes such as number of angles and number of equal faces
 |  |  |  |  |  |
| 1. Represents three-dimensional figures with nets, and interprets representations of two- and three-dimensional figures
 |  |  |  |  |  |
| 1. Uses definitions to identify lines, line segments, rays, and angles in two-dimensional figures
 |  |  |  |  |  |
| 1. Uses the coordinate plane and its conventions to reason and communicate about shapes and their properties and location
 |  |  |  |  |  |
| 1. Makes and justifies conjectures about geometric shapes and relations
 |  |  |  |  |  |
| 1. Statistics (10%)
 |  |  |  |  |  |
| 1. Understands basic concepts of statistics both conceptually and procedurally
 |  |  |  |  |  |
| 1. Identifies appropriate statistical questions and sampling procedures (e.g., randomness, size)
 |  |  |  |  |  |
| 1. Solves problems involving measures of center (mean, median, mode) and spread (e.g., range, variability)
 |  |  |  |  |  |
| 1. Determines how changes in data affect measures of center or range
 |  |  |  |  |  |
| 1. Describes a set of data in terms of center, spread, and shape and makes inferences from the data
 |  |  |  |  |  |
| 1. Knows how to represent and interpret data presented in various forms
 |  |  |  |  |  |
| 1. Constructs and interprets graphs that represent data (e.g., bar graphs, line graphs, histograms, double bar graphs, double line graphs, boxplots, lineplots/dotplots)
 |  |  |  |  |  |
| 1. Chooses appropriate graphs to display data (e.g., categorical, numerical)
 |  |  |  |  |  |
| 1. Identifies interests that might be served by given data and how a representation of data might be misleading
 |  |  |  |  |  |