| **Required Course Numbers** | | | | | | | | | | | | | | | |
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| **Test Content Categories** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **I. Specialized Mathematics Knowledge for Teaching** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **A. Instruction** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Knows how to evaluate explanations, justifications, and definitions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Identifies valid explanations of mathematical concepts (e.g., explaining why a mathematical idea is considered to be true), models, representations, or procedures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Evaluates or compares explanations and justifications for their validity, generalizability, coherence, or precision, including identifying flaws in explanations and justifications |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Determines the changes that would improve the validity, generalizability, coherence, or precision of an explanation or justification |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Evaluates definitions or other mathematical language for validity, generalizability, precision, usefulness in a particular context, or support of key ideas |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Knows how to evaluate problems, tasks, questions, examples, and procedures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Identifies problems, tasks, or questions that fit a particular structure, address the same concept, demonstrate desired characteristics, or elicit particular student thinking |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Identifies parallel problems that systematically vary in complexity in order to differentiate and provide appropriate challenge |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Sequences problems, tasks, or examples based on information about learning trajectories or standards progressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Evaluates the usefulness of problems, tasks, or examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Identifies examples or questions that support particular strategies or address particular student questions, misconceptions, or challenges with content |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Identifies nonexamples or counterexamples that highlight a mathematical distinction or demonstrate why a student conjecture is incorrect or partially incorrect |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Evaluates procedures for working with mathematics content in terms of validity, appropriateness, or rigor, or to identify special cases in which the procedure might be problematic |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3.Knows how to evaluate the use of representations and tools (e.g., models, manipulatives, technologies) to support a particular learning goal |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Evaluates representations (i.e., verbal, visual, physical, contextual, symbolic) in terms of validity, generalizability, usefulness for supporting students’ understanding, or fit to the concept, calculation, etc., to be represented |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Evaluates how representations (i.e., verbal, visual, physical, contextual, symbolic) have been used to show particular ideas, relationships between ideas, processes, or strategies |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Evaluates the appropriateness of technologies (e.g., virtual manipulatives, interactives, software) for supporting key ideas in different instructional settings (e.g., face-to-face, online, blended) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **B. Student Reasoning** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Knows how to evaluate student reasoning |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Identifies likely misconceptions about or partial understanding of particular mathematics content and full engagement in mathematical processes and practices |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Identifies how new mathematics content and practices can build on or connect to students’ prior knowledge |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Evaluates or compares student work (e.g., solutions, conjectures, explanations, justifications, generalizations, representations) in terms of validity, generalizability, coherence, or precision |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Evaluates student work to identify the use of a particular concept, idea, or strategy, and purposefully sequences the presentation of student work in class discussions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Evaluates whether a counterargument provides an accurate critique of a given student conjecture, explanation, justification, or generalization |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Identifies how a student’s reasoning would replicate across similar problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Identifies different pieces of student work that demonstrate the same or similar reasoning |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **C. Engagement in Mathematical Practices** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Knows how to create, maintain, and support opportunities for learners to develop their ability to engage in mathematical practices |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Identifies ways to create, maintain, or support opportunities for learners to make sense of problems and persevere in solving them |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Identifies ways to create, maintain, or support opportunities for learners to reason abstractly and quantitatively |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Identifies ways to create, maintain, or support opportunities for learners to construct viable arguments and critique the reasoning of others |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Identifies ways to create, maintain, or support opportunities for learners to model with mathematics |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Identifies ways to create, maintain, or support opportunities for learners to use appropriate tools strategically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Identifies ways to create, maintain, or support opportunities for learners to attend to precision |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Identifies ways to create, maintain, or support opportunities for learners to look for and make use of structure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. Identifies ways to create, maintain, or support opportunities for learners to look for and express regularity in repeated reasoning |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **II. Pedagogical Knowledge and Instructional Leadership (35%)** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **A. Pedagogical Knowledge for Teaching Mathematics** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understands how child, preadolescent, and adult learning and development affect the mathematical learning environment |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Identifies ways to draw on each and every learner’s mathematical strengths to create inclusive social learning contexts that engage each and every learner in discussions and mathematical explorations in order to motivate and extend learning opportunities that connect to each and every learner’s experience |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Identifies how to demonstrate and encourage equitable and ethical treatment of each and every learner and support each and every learner in achieving high expectations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses instructional formats (e.g., whole group, small group, partner, individual) skillfully and flexibly in support of learning goals and in consideration of various settings (e.g., face-to-face, online, blended) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Identifies ways to support the equitable learning of mathematics by embracing and purposefully incorporating diversities of the classroom and school—cultural, racial, ethnic, ability, linguistic, gender, socioeconomic, developmental, etc.; uses this knowledge to motivate and extend learning opportunities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Identifies ways to provide each and every learner with opportunities to make connections between mathematics and other content areas, everyday life, and the workplace |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Identifies ways to facilitate each and every learner’s engagement in productive struggle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understands equitable curriculum and assessment practices |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Identifies connections between mathematical concepts as well as the developmental progressions within these mathematical concepts |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses standards progressions and learning trajectories to organize and deliver instruction that is developmentally appropriate and responsive to individual learners |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses multiple strategies (e.g., asking probing questions, listening to learners) to assess mathematical knowledge and to understand thinking processes |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Determines the suitability of mathematics curricula and teaching materials (e.g., curricular resources, technology, manipulatives) and selects, uses, and adapts those materials appropriately for particular learning goals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Identifies the different formats, purposes, uses, and limitations of various types of assessment of student learning in order to choose, design, or adapt assessment tasks |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Uses the formative assessment process (administer a formative assessment task, analyze responses to the task, and determine appropriate actions based on this analysis) in order to inform teaching and benefit learning |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Analyzes formative and summative assessment results, makes appropriate interpretations, and communicates results to appropriate and varied audiences |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. Selects and uses strategies to provide and manage timely, targeted, and effective feedback (e.g., teacher to student, student to teacher, student to student, among peers) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **B. Instructional Leadership** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Knows how to provide instructional leadership in mathematics |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Identifies ways to promote and support a rigorous district instructional program based on research-supported best practices regarding curriculum, instruction, technology, and assessment |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Selects appropriate and effective methods for communicating professionally with educational stakeholders about students, curriculum, instruction, use of technology, and assessment |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Demonstrates knowledge of educational structures and policies that affect equitable access to quality mathematics instruction |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Identifies ways to advocate for the rights and needs of each and every learner to secure resources and promote academic advancement |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Identifies strategies for conferring and collaborating with stakeholders to develop, implement, evaluate, and improve mathematics programs (e.g., curriculum, instruction, professional development, parent/guardian training for at-home support for students) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Identifies professional development needs, and selects and uses strategies to plan, develop, implement, and evaluate professional development programs at the school or district level |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Identifies ways to use professional development (e.g., mentoring, coaching, peer-teaching, workshops) to facilitate appropriate research-supported, standards-based mathematics instruction and to promote the use of instructional methods supported by research |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. Identifies ways to support teachers in systematically reflecting and learning from practice (e.g., one-on-one observation, coaching cycle, video analysis, lesson study) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| i. Applies skills and strategies for mentoring, coaching, and consultation in the development, implementation, and evaluation of an effective mathematics program |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| j. Identifies differences among coaching moves (e.g., telling, direct guidance, invitational guidance) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| k. Identifies differences among roles on a continuum of instructional leadership (i.e., coach versus administrator) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| l. Identifies ways to translate research into practices that teachers can use |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| m. Selects and uses strategies to determine the impact of daily and annual contributions as an elementary mathematics specialist to mathematics teaching and learning, and uses efficacy data to advocate for the role |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |