| **Required Course Numbers** |
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| **Test Content Categories** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Number & Quantity and Algebra
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| 1. **Number and Quantity**
 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understands the structure of the real number system and how the basic operations on numbers in this system are performed |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Represents and solves word problems involving addition, subtraction, multiplication, and division of real numbers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Given operations on a number system, determines whether commutative, associative, and distributive properties hold |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Identifies whether the sum or product of rational and/or irrational numbers must be rational, must be irrational, or can be rational or irrational (e.g., the sum of two rational numbers must be rational, the product of two irrational numbers can be rational or irrational) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Solves problems involving number theory properties (e.g., prime, composite, prime factorization, even, odd, factors, multiples) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Uses proportional relationships to solve ratio, constant rate, and percent problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understands the properties of radicals and rational exponents |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Performs operations involving rational exponents |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses the properties of exponents to rewrite expressions that have radicals or expressions that have rational exponents |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses scientific notation to represent and compare numbers and to perform calculations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understands how to reason quantitatively and use units to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Chooses and interprets units consistently in formulas |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Chooses and interprets the scale and the origin in graphs and data displays |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Solves measurement, estimation, and conversion problems involving time, length, temperature, volume, and mass in standard measurement systems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Solves problems involving dimensional analysis (e.g., feet per second to miles per hour, feet per second to kilometers per hour) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Knows the structure of the complex number system and how basic operations with complex numbers are performed |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Performs operations with complex numbers, including conjugates |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Applies the commutative, associative, and distributive properties to complex numbers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **B. Algebra** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understands how to write algebraic expressions in equivalent forms |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses the structure of a polynomial or exponential expression to identify ways to rewrite it in an equivalent form (e.g., differences of squares, factoring, changing bases) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Understands how to rewrite algebraic expressions for specific purposes (e.g., factored form to find zeros, vertex form to find maxima or minima) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Rearranges formulas to solve for a specified variable |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Adds, subtracts, multiplies, and divides polynomials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Factors special polynomials over the complex numbers (e.g., ) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understands how to create equations and inequalities that describe relationships |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Creates equations and inequalities in one variable, uses them to solve problems, and graphs solutions on the number line |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Creates equations and inequalities in two or more variables, uses them to solve problems, and graphs the equations in two variables on the coordinate plane with appropriate labels and scales |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. In a modeling context, represents constraints by systems of equations and/or inequalities and interprets solutions as viable or nonviable options |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understands how varied techniques (e.g., graphical, algebraic, tabular) are used to solve equations and inequalities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Solves linear equations and inequalities in one variable, including equations with variable coefficients |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Solves quadratic equations with real coefficients that have complex solutions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses the method of completing the square to transform any quadratic equation in x into the equivalent form  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Solves equations using a variety of methods (e.g., graphing, factoring, using the quadratic formula) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Uses different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Graphs the solutions to a linear equation or inequality in two variables |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Justifies each step in solving an equation or inequality |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understands how varied techniques (e.g., graphical, algebraic, tabular) are used to solve systems of equations and inequalities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Solves a system consisting of two linear equations in two variables algebraically and graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Solves a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Finds the solutions of  approximately (e.g., uses technology to graph the functions, makes tables of values); includes cases where  and/or  are linear, polynomial, rational, absolute value, exponential, radical, or logarithmic functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Graphs the solution set to a system of linear inequalities in two variables |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understands the concept of rate of change of nonlinear functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Calculates and interprets the average rate of change of a function presented as a table of values, algebraically, or graphically over a given interval |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Recognizes and is able to extract and interpret information about a linear equation when it is presented in various forms (e.g., slope-intercept, point-slope, standard) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Calculates the intercepts of a line and interprets them in a modeling context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Calculates the slope of a line presented as a table of values, algebraically, or graphically, and interprets it in a modeling context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understands the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Applies the remainder theorem to find factors of polynomials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses factorization to identify zeros of polynomials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses zeros and factorization of a polynomial to sketch a graph of the polynomial and uses the graph to determine the zeros and the factorization of the polynomial |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Uses a variety of techniques to find and analyze the zero or zeros (real and complex) of polynomial functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Understands how to rewrite rational expressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Rewrites simple rational expressions in an equivalent form |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Adds, subtracts, multiplies, and divides rational expressions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Understands how to justify the reasoning process used to solve equations, accounting for potential extraneous solutions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Solves simple rational and radical equations in one variable, accounting for potential extraneous solutions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| II. Functions and Calculus |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **A. Functions** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understands functions and function notation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Determines whether a relation is a function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Evaluates functions and interprets statements that use function notation in terms of a context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Determines the domain and range of a function from a function rule (e.g.,  graph, set of ordered pairs, or table |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understands how function behavior is analyzed using different representations (e.g., graphs, mappings, tables) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. For a function that models a relationship between two quantities, interprets key features of graphs and tables (e.g., increasing/ decreasing, maximum/ minimum, discontinuities, end-behavior) in terms of the quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Given a verbal description of a function, sketches graphs that show key features of that function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Graphs functions (e.g., linear, quadratic, exponential, piecewise, absolute value, step, radical, polynomial, rational, logarithmic, trigonometric) defined by an expression and identifies key features of the graph |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Writes a function that is defined by an expression in different but equivalent forms to reveal different properties of the function (e.g., zeros, extreme values, symmetry of the graph) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Interprets the behavior of exponential functions (e.g., growth, decay) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Determines whether a function is odd, even, or neither and whether the graph of the function has any symmetries |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Compares properties of two functions each represented in a different way (e.g., as a table of values, algebraically, graphically, or by verbal descriptions) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. Recognizes and is able to extract information about a quadratic function when it is presented in various forms (i.e., standard, vertex, factored) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| i. Converts among various forms of quadratic equations (i.e., standard, vertex, factored) using methods such as factoring and completing the square |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understands how functions and relations are used to model relationships between quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Writes a function that relates two quantities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Determines an explicit expression or a recursive process that builds a function from a context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Writes arithmetic and geometric sequences both recursively and with an explicit formula and uses them to model situations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Translates between recursive and explicit forms of arithmetic and geometric sequences |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understands how new functions are obtained from existing functions (e.g., compositions, transformations, inverses) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Describes how the graph of is related to the graph of , is related to the graph of where   or  for specific values of *k* (both positive and negative) and finds the value of *k* given the graphs |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Given that a function *f* has an inverse, finds values of the inverse function from a graph or a table of *f* |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Interprets the meaning of an inverse function in a modeling context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Given a noninvertible function, determines the largest possible domain of the function that produces an invertible function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Given a relation, finds its inverse and determines if its inverse is a function and writes an expression for the inverse function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Uses the inverse relationship between exponential and logarithmic functions to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Combines standard function types |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| h. Analyzes the domain of functions created by combining functions using arithmetic operations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| i. Composes functions presented as tables of values, algebraically, or graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| j. Analyzes the domain of functions resulting from composition |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| k. Uses composition to express the relationship between a function and its inverse |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Understands that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Identifies situations in which one quantity changes at a constant rate per unit interval relative to another |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Identifies situations in which a quantity using arithmetic operations grows or decays by a constant percent rate per unit interval relative to another |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Constructs linear and exponential functions, including arithmetic and geometric sequences, given a graph, table of values, a set of ordered pairs, or a description of a relationship |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Observes that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratic ally, or (more generally) as a polynomial function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Interprets the parameters in a linear or exponential function in terms of a context (e.g.,  ) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understands how to use logarithms to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Applies the properties of logarithms to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Expresses the solution to an exponential equation with base b as a logarithm (e.g., , ) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses technology to evaluate logarithms that have any base |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understands the relationship between points on the unit circle and the values of trigonometric functions for any given angle measure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Converts between degree measure and radian measure |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Identifies the reference angle for a given angle and the relationship between the trigonometric values of an angle and its reference angle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Finds the values of trigonometric functions of any angle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Uses the unit circle to explain symmetry and periodicity of trigonometric functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Understands how periodic phenomena are modeled using trigonometric functions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Chooses trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses inverse functions to solve trigonometric equations that arise in modeling contexts and interprets them in terms of the context |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Understands how to solve trigonometric, logarithmic, and exponential equations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Solves trigonometric, logarithmic, and exponential equations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **B. Calculus** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understands the meaning of a limit of a function and how to calculate limits of functions and conditions when the limit does not exist |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Solves limit problems using properties of limits, where all limits of the individual functions exist at the value that x is approaching |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Determines limits to a fixed value and interprets the ‘limit graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Determines one-sided limits to a fixed value, from both left and right, interprets the limit graphically, and uses it to determine if the limit to the value exists |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Computes limits to infinity or negative infinity and interprets the result graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Identifies limits that do not exist for functions presented algebraically or graphically |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understands the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Given the graph of a function and a point on that graph, knows the relationship between the derivative of the function at that point, the slope of the tangent to the graph at that point, and the succession of slopes of secant lines connecting  to as *x* approaches *a* |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understands what it means for a function to be continuous at a given point and knows the relationship between continuity and differentiability |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Applies the three steps  exists,  exists, and  that are part of the definition of what it means for a function to be continuous at  to verify whether a given function is continuous at a given point |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Gives examples of functions that are continuous at  but not differentiable at and explains why |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understands how and when to use standard differentiation and integration techniques |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses standard differentiation techniques |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses standard integration techniques, including both definite and indefinite integrals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses the relationship between the position, velocity, and acceleration functions to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understands how to analyze the behavior of a function (e.g., extrema, concavity, symmetry) and understands how to use integration to compute area |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses the first and second derivatives to analyze the graph of a function |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Matches graphs of functions with graphs of their derivatives or accumulations based on the second part of the fundamental theorem of calculus |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses integration techniques to compute area |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| III. Geometry |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **A. Geometry** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Knows the properties of lines (e.g., parallel, perpendicular, intersecting) and angles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Solves problems involving parallel, perpendicular, and intersecting lines |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Knows the properties of triangles, quadrilaterals (e.g., parallelogram, rectangle, rhombus), and other polygons |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Determines whether given side lengths or angle measures would produce a triangle (e.g., triangle inequality theorem) and classifies triangles by their sides or angles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Solves problems involving special triangles (e.g., isosceles, equilateral, right,) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses the definitions of median, midpoint, and altitude to solve problems involving triangles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Identifies geometric properties of various quadrilaterals and the relationships among them (e.g., parallelogram, rectangle, rhombus) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Solves problems involving sides, angles, or diagonals of polygons |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understands transformations in the plane |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses rigid motions (e.g., translations, rotations, reflections) to transform figures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses dilations to transform figures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Applies properties of rigid motions (e.g., rigid motions preserve distance and angle measure) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Applies properties of dilation transformations (e.g., dilation transformations preserve angle measure but not distance) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Identifies a sequence of transformations that maps a preimage onto an image |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Given a figure, describes the transformations that map the figure onto itself, including reflection over a line of symmetry |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Represents translations using vector notation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understands congruence and similarity |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Determines whether two figures are congruent using triangle congruence theorems (e.g., ASA, SAS, SSS) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Determines whether two figures are similar using triangle similarity theorems (e.g., AA criterion) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Determines whether two figures are congruent by directly mapping one figure onto another using a sequence of one or more rigid motions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Determines whether two figures are similar by directly mapping one figure onto another using a sequence of one or more transformations (dilations and/or rigid motions) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Uses congruence and similarity to solve problems involving unknown side lengths or angle measurements in two-dimensional and three-dimensional figures |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Knows how to prove geometric theorems such as those about lines, angles, triangles, and parallelograms |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Solves problems involving proofs of theorems about lines and angles (e.g., vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are equidistant from the segment’s endpoints) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Solves problems involving proofs of theorems about triangles (e.g., measures of interior angles of a triangle sum to ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point; a line parallel to one side of a triangle divides the other two sides proportionally; the Pythagorean theorem proved using triangle similarity) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Solves problems involving proofs of theorems about parallelograms (e.g., opposite sides are congruent; opposite angles are congruent; the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Identifies whether geometric proofs are valid (e.g., direct proofs, counterexamples) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understands how trigonometry is applied to triangles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses the relationship between the sine and cosine of complementary angles to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses trigonometric ratios and the Pythagorean theorem to solve for side lengths and angle measures of right triangles in geometric or applied problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses the values of trigonometric functions of special angles (e.g., ) to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Applies the Law of Sines and the Law of Cosines to find unknown measurements in triangles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Understands how to apply theorems about circles |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Solves problems involving circumference and area of a circle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Solves problems involving lengths of arcs and areas of sectors |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Solves problems involving measures of inscribed angles, central angles, circumscribed angles, and arcs |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Uses properties of lines in a circle to solve problems (e.g., chords, secants, tangents, radii, diameters) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Identifies and uses the geometric description of a circle as the set of points for which the distance from a point to a fixed point (the center) is constant |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Determines the equation of a circle given the center and radius of the circle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Finds the center and radius of a circle given by an equation of the circle in any form |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Understands how to use coordinate geometry to describe properties of geometric objects |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses coordinate geometry to represent and identify the properties of geometric shapes and to solve problems (e.g., Pythagorean theorem, perimeter of a polygon, area of a rectangle) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Determines the distance between two points |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Finds the point on a directed line segment between two given points that partitions the segment in a given ratio |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Uses the slope criteria for parallel and perpendicular lines to solve geometric problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9. Knows how to solve problems involving perimeter and area of polygons |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Calculates and interprets perimeter and area of polygons that can be composed of triangles and quadrilaterals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Calculates changes in perimeter and area as the dimensions of a polygon change |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10. Knows how to solve problems involving solids |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Calculates and interprets surface area and volume of solids (e.g., prisms, pyramids, cones, cylinders, spheres), including in real-world situations |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Calculates changes in surface area and volume as the dimensions of a solid change |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Identifies the shapes of two-dimensional cross sections of three-dimensional objects and identifies three-dimensional objects generated by rotations of two-dimensional objects |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Uses two-dimensional representations (e.g., nets) of three-dimensional objects to visualize and solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| IV. Statistics & Probability |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **B. Statistics & Probability** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1. Understands how to make inferences and justify conclusions from samples, experiments, and observational studies |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses statistics to make inferences about population parameters based on a sample from that population |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Identifies the purposes of and differences among sample surveys, experiments, and observational studies and explains how randomization relates to each |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses data from a sample survey to estimate a population mean or proportion |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Uses data from a randomized experiment to compare two treatments |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2. Understands how to summarize, represent, and interpret data collected from measurements on a single variable (e.g., boxplots, dot plots, normal distributions) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Represents and interprets data with plots on the real number line (e.g., dot plots, histograms, boxplots) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Computes the center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) for a data set |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Uses statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Interprets differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3. Understands how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Summarizes and interprets categorical data for two categories in two-way frequency tables (e.g., joint, marginal, conditional relative frequencies) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Identifies possible associations and trends in the data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Represents and interprets data for two quantitative variables on a scatterplot and describes how the variables are related |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4. Understands how to create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses technology to fit a function to data (i.e., linear regression) and determines a linear correlation coefficient |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses functions fitted to data to solve problems in the context of the data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Assesses the fit of a function by plotting and analyzing residuals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Interprets the slope and the intercept of a regression line in the context of the data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Interprets a linear correlation coefficient |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Distinguishes between correlation and causation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5. Understands the concept of independence and understands how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Describes events as subsets of a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Determines and interprets when two events are independent |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Identifies and applies the concepts of conditional probability and independence |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| d. Calculates probabilities of simple and compound events |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| e. Constructs and interprets two-way frequency tables of data when two categories are associated with each object being classified; uses the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| f. Applies the addition rule,  and interprets it in terms of a given model |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| g. Applies the general multiplication rule in a uniform probability model,  and interprets it in terms of a given model |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6. Understands how to find probabilities involving finite sample spaces and independent trials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Uses the fundamental counting principle to find probabilities involving finite sample spaces and independent trials |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses counting techniques (e.g., permutations, combinations) to solve problems |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7. Knows how to make informed decisions using probabilities and expected values |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Interprets a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and finds the expected value |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Interprets a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and finds the expected value |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Weighs the possible outcomes of a decision by assigning probabilities to outcomes and finding expected values |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8. Understands normal distributions |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a. Identifies whether data sets are normally distributed based on their shape |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| b. Uses the mean and standard deviation of a normal distribution to interpret population percentages |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| c. Estimates and interprets areas under the normal curve |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |